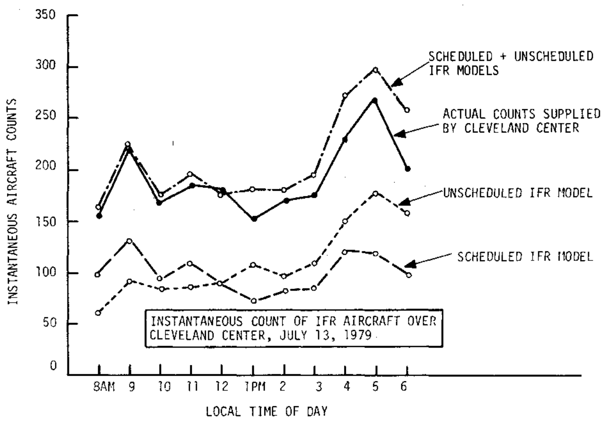

Fig. 1 Instantaneous count of IFR aircraft over Cleveland center, July 13, 1979.

number of aircraft in the air, allows for the development of an efficient model. The model is efficient in the sense that it is unnecessary to continuously collect and automate a potentially large amount of data, thereby greatly reducing computer costs incurred in processing those data. Additional air traffic analyses are also facilitated.

Comparison of Model Predictions with Field Data

On July 13 (Friday), 1979, Cleveland center recorded the number of actively controlled aircraft as a function of time of day. (This recording also included a small number of controlled non-IFR aircraft.) Cleveland center, which controls the airspace over parts of Ohio, Michigan, New York, Pennsylvania, and West Virginia, is one of the busiest centers. The sum of the models' predicted counts of scheduled and unscheduled aircraft was compared with the recorded counts for the above data for the times, 8 a.m.-6 p.m., on the hour.

At the time the comparisons were made, unscheduled aircounts could be predicted for any day in April 1978. Therefore seasonal adjustment factors for GA activity based on historical data[4] were applied to these counts, so that they would be representative of counts on a typical Friday in July 1979.

Figure 1 shows predicted IFR counts and actual counts over Cleveland center. (Data points are connected by lines as a visual aid.) Similar comparisons were made for Houston, Kansas City, and New York centers (one day for each center in the summer of 1979, although the day of the week was different in each case). All comparisons between predicted and measured aircounts show good agreement in terms of small numerical differences in the amplitudes and also similar harmonic content. The largest difference occurs at the peaks and is about 10%. This agreement is sufficient for system planning purposes.[3] It is planned to extend these comparisons to other centers for additional days and seasons of the year, and to quantify the (statistical) differences between measured and predicted counts.

Acknowledgment

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Unrestrained Aeroelastic Divergence in a Dynamic Stability Analysis

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A QUASISTATIC formulation of the divergence problem for a vehicle free to plunge is given in Ref. 1 and has been generalized to multiple rigid body degrees of freedom in Ref. 2. The restrained divergence problem has been investigated by two dynamic stability methods in Ref. 3: the transient method of Ref. 4 and the British flutter method.[5-7] It is the purpose of this Note to investigate unrestrained divergence by the dynamic stability methods of Ref. 3 in order to determine the validity and accuracy of the quasistatic approximations of Refs. 1 and 2.

The first two examples of Ref. 3 are reconsidered here with the bending and torsion springs attached to a mass (fuselage), instead of to ground, free to move vertically. The characteristics of the two examples are the same except for the center of gravity location: in example 1 it is at 37% chord, and in example 2 it is at 45% chord. Both examples have the aerodynamic center at 25% chord ($\xi = 0.25$ in Ref. 4) and the elastic axis at 40% chord. The remaining parameters for the analysis at sea level include a chord of 6.0 ft, a radius of gyration of 1.5 ft about the elastic axis, a mass ratio $\mu = 20.0$, the uncoupled bending and torsion frequencies of 10.0 rad/s and 25.0 rad/s, respectively, and equal structural damping coefficients $g = 0.03$ in both modes. The lift curve slope is the theoretical incompressible value of 2π and the downwash is matched at the ¾-chord location ($r = 1.0$ in Ref. 4). The transient aerodynamic constants are $\alpha_1 = 0.165$, $\alpha_2 = 0.335$, $\beta_1 = 0.041$, $\beta_2 = 0.320$. In addition, the "fuselage" mass is assumed to be equal to the airfoil mass so that the mass ratio of Ref. 2 is $r = 0.50$.

The results of the transient solution for example 1 (with the forward center of gravity at 37%) are shown by the curves in Fig. 1. Divergence (from the aerodynamic lag root corresponding to $\beta_1 = 0.041$) occurs at 232.1 ft/s and flutter (from the torsion root) occurs at 294.3 ft/s. The stable solution for the second lag root (corresponding to $\beta_2 = 0.320$) is not shown. In contrast with Ref. 3, in which divergence was nonoscillatory, we see in Fig. 1 that the unrestrained divergence is *oscillatory* with a frequency of 1.189 Hz. We seem to have a semantic problem now. An unrestrained vehicle does not diverge statically but dynamically: shall we call this instability divergence or flutter? The oscillatory characteristic of unrestrained divergence has been calculated before in studies of the oblique-wing aircraft[8-11] but in those studies it was described as low frequency flutter. The present authors will continue to use the terminology of "dynamic divergence" because the instability finds its origin in a tendency to static divergence. The terminology "body freedom flutter" has also been used to describe dynamic divergence of forward-swept-wing aircraft,[12] but that does not seem appropriate since Gaukroger[13] has used those terms to describe a coupled body-pitch/wing-bending flutter in which the body has a relatively low value of pitching moment of inertia; the examples here have no rigid body degree of freedom in pitch.

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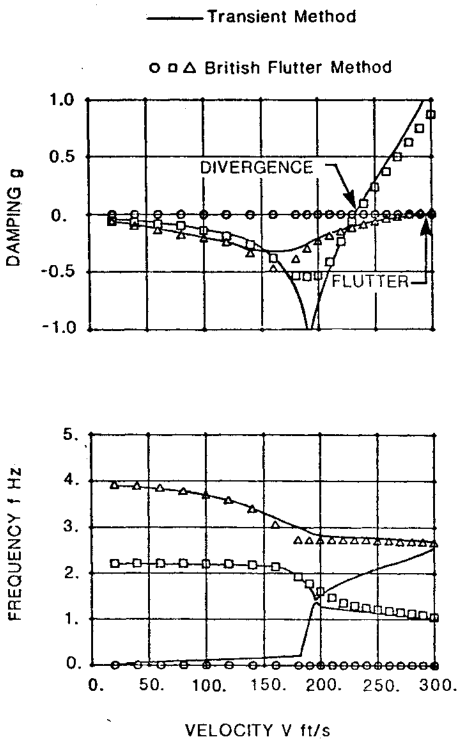

Fig. 1 Dampings and frequencies data of a 2 DOF airfoil with forward center of gravity plus a fuselage plunge degree of freedom.

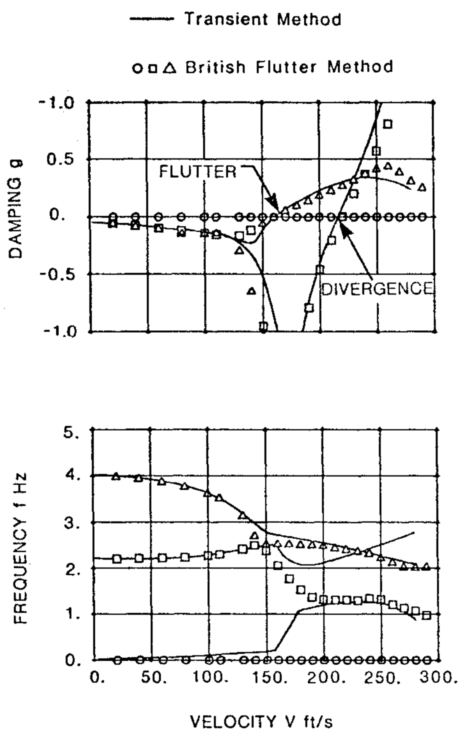

Fig. 2 Dampings and frequencies data of a 2 DOF airfoil with aft center of gravity plus a fuselage plunge degree of freedom.

The quasistatic unrestrained divergence speed of Refs. 1 and 2 [and Ref. 14, Eq. (12)] is 228.2 ft/s for example 1 and is lower than the value of 232.1 ft/s by 1.7%. The difference is caused by the aerodynamic lag effects and virtual masses in the transient aerodynamic formulation.

The results of the transient solution for example 2 (with the aft center of gravity at 45%) are shown by the curves in Fig. 2. Flutter is found at a velocity of 158.4 ft/s with a frequency of 2.768 Hz (the flutter frequency of example 1 was 2.664 Hz) and "dynamic divergence" is found at a velocity of 215.0 ft/s with a frequency of 1.184 Hz. For this second example, Refs. 1, 2, and 14 yield a quasistatic unrestrained divergence speed of 200.4 ft/s which is lower than the 215.0 ft/s by 6.8%.

The results of the British flutter method (called the PK-method in NASTRAN®[7]) are shown by the data symbols in Figs. 1 and 2. The calculations were made using MSC/NASTRAN with the W.P. Jones approximation to the Theodorsen function for consistency with the transient method. In the figures, the circular symbols show the rigid body plunge data, the square symbols show first (at low speeds) the bending data and then (at higher speeds) the aerodynamic lag data, and the triangular symbols show the torsion data. The transition from the bending root to the aerodynamic lag root is gradual in these two examples as compared to the corresponding examples in Ref. 3 where the transition was abrupt. The reason for the different behavior is found in the finite frequency of the "dynamic divergence" of an unrestrained system, whereas divergence of a restrained system occurs at zero frequency.

The purposes of this limited investigation were to determine the validity and accuracy of the quasistatic unrestrained divergence analysis. Its validity is now questionable because the solution by dynamic stability methods has been found to be qualitatively different, i.e., it is oscillatory rather than quasistatic. However, its accuracy is not unreasonable: 1.7% error in the first example, and 6.8% error in the second. More general conclusions must await an evaluation of the quasistatic approximation on more practical configurations including, at least, the additional rigid body degree of freedom of pitch. The British flutter method will provide a reliable means for such an evaluation.

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